The article presents mathematical models for determining the bubble point diameter of the pores of meshes with micron sizes square cells of plain and full twill weaving. Analyzed are mathematical models developed by a number of researchers to determine the bubble point pore diameter of the meshes and their point pore pressure. The analysis of the mathematical models showed that they were developed on the basis of a number of assumptions that, to varying degrees, take into account the spatial shape of the mesh cells and the physicochemical processes when the free surface through the mesh. This undoubtedly affects the accuracy of analytical calculations that are based on the use of these models. In the presented work, mathematical models have been developed that take into account the complex spatial shape of the wires from which the mesh is made. Mathematical models have been developed for plain and twill weaving meshes. When developing mathematical models, it was assumed that the free surface is formed on a hole with elliptical edges. In this case, it is assumed that the points of contact of the free surface of a plain weave mesh correspond to the most distant points of the projection of the mesh cell. For full twill meshes, taking into account the structure of their weaving, it is assumed that the free surface is formed on two adjacent cells. The influence of the contact angle between the mesh surface and the working fluid was taken into account by analyzing the spatial shape of the mesh wires. The developed models make it possible to take into account the effect of liquids with a contact angle from zero to 90 degrees. Simplified mathematical models of the meshes have also been developed for the case of perfect wetting of the mesh surface with a liquid, when the contact angle is zero. The obtained mathematical models can be used in the design of capillary phase separators of liquid acquisition devices in zero gravity of spacecraft, as well as in the calculation of capillary devices of various technical systems, such as heat exchangers, gas-liquid mixers, chemical reactors, bubble filters, etc.

Keywords: LIQUID ACQUISITION DEVICE, MICROGRAVITY, PHASE SEPARATION, BUBBLE POINT, MESH SCREEN, WEAVE TYPE
Математическая модель определения эквивалентного капиллярного диаметра пор сеток с квадратными ячейками

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Аннотация. В статье приведены математические модели для определения эквивалентного капиллярного диаметра пор сеток с квадратными ячейками микронных размеров полотняного и саржевого типа плетения. Проведен анализ математических моделей, разработанных рядом исследователей, для определения эквивалентного капиллярного диаметра пор сеток и их капиллярной удерживающей способности. Анализ математических моделей показал, что они разработаны исходя из ряда допущений, которые в различной степени учитывают пространственную форму ячеек сеток и физико-химические процессы при прохождении через них поверхности раздела фаз. Это несомненно влияет на точность аналитических расчетов, которые основаны на использовании этих моделей. В представленной работе разработаны математические модели, которые учитывают сложную пространственную форму проволок, из которых изготовлена сетка. Математические модели разработаны для сеток полотняного и саржевого типа плетения. При разработке математических моделей принято допущение, что поверхность раздела фаз формируется на отверстии с краями эллиптической формы. При этом предполагается, что точки контакта поверхности раздела фаз на сетке полотняного типа плетения соответствуют наиболее удаленным точкам проекции ячейки сетки. Для сеток саржевого типа плетения, учитывая структуру их плетения, предполагается, что поверхность раздела фаз формируется на двух соседних ячейках. Влияние краевого угла смачивания между поверхностью сетки и рабочей жидкостью учитывалось с помощью анализа пространственной формы проволок. Разработанные модели позволяют учитывать влияние жидкостей с краевым углом смачивания от нуля до 90 градусов. Также разработаны упрощенные математические модели сеток для случая полного смачивания поверхности жидкостью, когда краевой угол смачивания равен нулю. Полученные математические модели могут быть использованы при проектировании капиллярных фазоразделителей систем обеспечения сплошности компонентов топлива в невесомости космических летательных аппаратов, а также при расчете капиллярных устройств различных технических систем, таких как теплообменники, газожидкостные смесители, химические реакторы, пузырьковые фильтры и т.д.

Ключевые слова: СИСТЕМА ОБЕСПЕЧЕНИЯ СПЛОШНОСТИ ТОПЛИВА, МИКРОГРАВИТАЦИЯ, РАЗДЕЛЕНИЕ ФАЗ, КАПИЛЛЯРНОЕ ДАВЛЕНИЕ, СЕТЧАТЫЙ ЭКРАН, ТИП ПЛЕТЕНИЯ.

Introduction
To perform orbital maneuvers, such as launching spacecraft into target orbits [1], as well as transitioning to a graveyard orbit, or entering to dense layers of the atmosphere [2], upper stages of launch vehicles must be able to repeatedly launch the propulsion system in weightlessness. To ensure this possibility, capillary liquid acquisition device have become widespread [3, 4]. Such systems, due to surface tension forces, hold in the intake device area, part of the propellant required to start the engine. The main working element of such systems is a screen made of a mesh having micron-sized cells [4] (fig. 1).

The most important design characteristic of mesh screen is bubble point pressure (BP), which is determined by the maximum static pressure drop at which the liquid-gas free surface (FS) remains stable and prevents the penetration of gas [4, 6].

![Figure 1 – Weaving structure of plain (a) and full twill (b) mesh [5]](image)

Under the action of an external pressure drop, FS moves inside the mesh cell and bends
to achieve equality with the capillary pressure $p_c$, which occurs as a result of surface tension forces. The value of the capillary pressure directly depends on the surface tension of the liquid $\sigma$, the curvature FS and determined by the Young-Laplace equation [4]:

$$p_c = \sigma \cdot \left( \frac{1}{R_1} + \frac{1}{R_2} \right),$$

(1)

where $R_1$ and $R_2$ – principal radii of curvature FS.

When FS is contact with the mesh, a contact angle $\theta$ occurs, which affects the FS shape and depends on the intermolecular forces of interaction of liquid particles and the mesh material [7].

For example, for a cylindrical capillary, taking into account the contact angle, equation (1) takes the form [4, 6]:

$$p_c = \frac{4 \cdot \sigma}{d_c} \cos(\theta),$$

(2)

where $d_c$ – bubble point diameter.

The spatial structure of the mesh (fig. 1) has a more complex shape then the cylindrical capillary. To determine the principal radii of curvature FS mathematical model must take into account the mesh weaving structure (plain, twill), geometric characteristics of its cells (mesh opening size $a_{ob}$, wire diameter $d_w$) and the contact angle its surface with liquid $\theta$.

Consider the mathematical models used to determine the BP and $d_c$ of the mesh pores.

In [8] proposed to replace the complex spatial structure of the mesh with a perforated plate of zero thickness with round holes.

In [9] the mesh considered as a perforated plate, which has a thickness equal to the thickness of the mesh. It is assumed that such a plate has a pore $d_c$ and a coefficient of hydraulic resistance, as in its mesh prototype.

In [10] describes a mathematical model in which the weave structure of the mesh is described by a toroidal surface, the parameters of which correspond to the parameters of the mesh.

In [11] it is stated that the value of capillary pressure in the square cell of the mesh can be in the range from the value of capillary pressure in a cylindrical capillary to the value of such pressure between two parallel plates. The diameter of the hole and the distance between the plates are taken accordingly the same.

To determine the capillary pressure in a cell of any shape in [4] it is proposed to use as $d_c$ the value of the hydraulic diameter of such cell, which is determined by the equation [12]:

$$d_g = \frac{4 \cdot S_o}{P_o},$$

(3)

where $S_o$ – the cross-sectional area of the mesh cell hole; $P_o$ – wetted perimeter of the mesh cell hole.

In [13] as $d_c$ it is proposed to take the diameter of the circle described around the hole of the mesh cell, then:

$$p_c = \frac{2\sqrt{2} \cdot \sigma}{a_{ob}}.$$  

(4)

In [14] as a $d_c$ it is offered to use the value of diameter which is defined by the equation:

$$d_c = \frac{2 \cdot \sqrt{S_o}}{\sqrt{\pi}}.$$  

(5)

In [15] the capillary pressure inside a square-shaped hydrophilic capillary with flat walls is determined by analyzing the distribution of residual fluid, then:

$$p_c = \begin{cases} 2 \sigma \cdot \frac{\cos(\theta)}{a_{ob}} \left( \frac{\pi}{4} - \theta + \frac{1}{2} \sin(\theta) \right), & \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \end{cases}$$

(6)

The above mathematical models have different degrees of similarity to the real geometry of the mesh, as well as different degrees of consideration of the physic process of passing FS through the mesh, which significantly affects the accuracy of calculations. As a result, experimental methods are most often used to determine BP and $d_c$ pores of the mesh.

A large number of works have been devoted to the experimental determination of $d_c$ and BP of meshes with square cells [6, 11, 16, 17, 18]. However, the determination of BP, for each type of mesh, material and liquid by experiment, is associated with high material and time costs.
Thus, the creation of a mathematical model that will allow the most accurate and fast determination of mesh cells $d_c$ analytically, taking into account its real geometry and the contact angle, is an urgent task.

**Formulation of the research problem**

The aim of the work is to develop a mathematical model for determining the bubble point diameter $d_c$ of pores meshes with micron sizes square cells of plain and twill weaving, taking into account their spatial structure and contact angle.

To achieve the aim of the work the following tasks are set:

- to develop mathematical model of FS formation in the mesh cell, taking into account its spatial shape and the contact angle the surface with liquid;
- to develop a mathematical model of FS formation on the asymmetric contour of the mesh cell;
- to develop the recommendations for application of mathematical models in the design of mesh phase separators.

**Solution of the problem**

Consider the spatial weaving structure of meshes with square cells of micron size (fig. 1). The warp and the weft wires overlap each other depending on the type of weaving so that the projection of the hole of each cell has a square shape. However, the FS passing through the mesh is not a plane and has a curvilinear shape. Thus, in a mathematical model it is must to take into account such a spatial structure.

In plain weave meshes (fig. 1a) each warp wire (or weft) overlaps the following weft wire (or warp). In full-twill weave meshes (fig. 1b), each warp wire (or weft) overlaps the next two weft wires (or warp) at once. However, we can observe that the basic geometric dimensions of the mesh of both types are identical.

The main geometric characteristics of the meshes are the opening size $a_{ob}$ and the wire diameter $d_w$ [5]. The wires of mesh are curved in a plane perpendicular to the projection plane of the mesh (fig. 1). Given the geometric ratios, the angle of wire inclination $\beta_w$ is define as:

$$\beta_w = 2 \arctg \left[ \frac{1}{3} \left( \frac{a_{ob}}{d_w} + 1 - \sqrt{\left( \frac{a_{ob}}{d_w} \right)^2 - 3} \right) \right]. \quad (7)$$

We can assume that in the cell of plain weave mesh (fig. 2a) FS is forms in a square hole. The minimum radius of FS curvature, at which it remains stable, depends on the distance between the maximum opposite points of contact of the mesh surface with FS, and the hole is symmetrical $a_1 = a_2$ (fig. 2a).

The position of the wires in the cell of full-twill weave mesh (fig. 1b) is such that above of one warp wire (or weft) is always 2 weft fibers (or warp). Thus, we can assume that the total FS can forms on two adjacent cells (fig. 1b). Then the hole of the cell of twill weaving mesh can imagines as an elongated rectangle. The width of such a rectangle corresponds to the minimum distance between the wires of the mesh

$$a_1 = a_{ob}, \quad (8)$$

and length of the rectangle side is determined by the projection of the double length of hole and one wire

$$a_2 = 2 \cdot a_{ob} + d_w. \quad (9)$$

When the characteristic dimensions of cell $a_1 < a_2$, as in the latter case, taking into account (1), FS can remain stable even at $2R_1 > a_1$ (fig. 3).
Substituting (1) into (2), after transformations, we obtain the equation for determining the $d_c$ of hole complex shape:

$$d_c = \frac{4}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}.$$  \hfill (10)

Equation (10) shows that to determine $d_c$ we must to establish the value of principal radii of curvature FS at a pressure drop equal to BP. In the general case, for any shape of a hole, it is not possible to analytically determine the principal radii of curvature FS at its upper point. However, with a number of assumptions and simplifications, an analytical or numerical solution of this task is possible.

Let us determine the functions of the principal radii of curvature from a single variable, for example, from the height of the bubble $h_b$, which is plotted from a given zero plane. Then, taking into account (10), $d_c$ will be define as a function of bubble height from the condition:

$$d_c(h_b) = \frac{4}{\left(\frac{1}{R_1(h_b)} + \frac{1}{R_2(h_b)}\right)} \rightarrow \min.$$  \hfill (11)

The values of the principal radii of curvature corresponding to the maximum value of capillary pressure at which FS remains stable inside the mesh cell will be at the point where function (11) reaches a minimum. Thus, the key factor in finding the $d_c$ of the any shape hole is the establish the functions of the principal radii of curvature from the height of the bubble $R(h_b)$. In some cases, it is possible to obtain an analytical solution of equation (10).

Consider the equilibrium position of the FS inside the mesh cell made of hydrophilic to the working fluid material with different variants of the cross-sectional shape of the hole (fig. 4).

In the process of moving the FS through a cylindrical hole with sharp edges, there are several characteristic equilibrium positions of the FS (fig. 4a). When the gas comes into contact with the surface of phase separator, the FS with a curvature is formed, which depends on the pressure difference between the gas and the liquid. FS is in equilibrium position 1 at the edge of the phase separator on the gas side. As the pressure difference increases, the radius of curvature FS decreases and when reached

$$R = \frac{a_c}{2 \cdot \cos(\theta)},$$  \hfill (12)

FS will already be inside hole 2 and will move to the edge on the liquid side [9].

With a further increase in the pressure difference is a decrease in the radius of curvature FS, which is in a stable position 3 at the edge of the phase separator, to the value.
A further increase in the pressure difference leads to an increase of the FS curvature radius $R$ and, accordingly, to a decrease in capillary pressure. As a result, this leads to uncontrolled growth of the bubble and loss of functionality of the phase separator [9].

We can assume that the BP of the cylindrical hole with sharp edges corresponds to the position of FS 3 in fig. 4a. However, if the edge of the hole is rounded, the FS may move to the surface of the plate after passing through the hole and begin to grow uncontrollably there. This will cause the phase separator to become inoperable.

In the case of complete wetting of the mesh material with liquid, when the contact angle $\theta = 0^\circ$, for the calculation of BP, we can assume that the thickness of the phase separator is zero.

Assuming that FS is forms at the hole of zero thickness, FS can considered as a segment of a circle. Then the dependence of the radius of curvature FS on the height of the bubble $R(h_b)$ we must to solve a system of equations with respect to one common variable (angle $\varphi$):

$$R(\varphi) = \frac{a_c + d_f \cdot [1 - \cos(\varphi)]}{2 \cdot \cos(\theta - \varphi)};$$

$$h_b(\varphi) = \frac{d_f}{2} \sin(\varphi) + R(\varphi) \cdot [1 - \sin(\theta - \varphi)].$$

In the general case, the dependence $R(h_b)$ cannot be obtained analytically. The solution exists only for some defined contact angles (eg $\theta = 0^\circ, \pi/2, \pi$). To determine the dependence for any contact angle $\theta$, we must to numerically solve equations (15) and (16) with respect to the angle $\varphi$.

After analysis of the mesh cell geometry (fig. 4b), equations (15), and (16), we can distinguish two critical stable positions FS, at which the minimum value of the radius of curvature FS (positions 2 and 3 in fig. 4b), and so thus, the maximum value of capillary pressure.

At position 2 (fig. 4b) FS is in the center of the hole and the radius of curvature will be identical to equation (12).

In position 3 (fig. 4b), the point of contact FS with the surface of the hole is in such a position that the FS cross section is a semicircle. Then the radius of curvature FS is determined by the equation:

$$R = \frac{a_c + d_f \cdot [1 - \cos(\theta)]}{2}. \quad (17)$$

Due to the wavy shape of the mesh wire, its cross section have an elliptical contour.

When the FS moves through the hole with elliptical edges (fig. 4c), taking into...
account the contact angle, there are also many equilibrium positions. The movement of FS through such a hole is similar to the movement through a hole with round edges. The hole with round edges is a special case of an elliptical hole. To define the dependence of the radius of curvature FS on the height of the bubble \( R(h_b) \) requires the solution of the following system of equations:

\[
\begin{align*}
\hspace{1cm} h_b(\varphi) &= \rho(\varphi) \cdot \sin(\varphi) + \\
&+ R(\varphi) \cdot \left[ 1 - \sin \left( \theta - \frac{\pi}{2} \right) - \psi(\varphi) \right]; \\
R(\varphi) &= \frac{L_f}{2} - \rho(\varphi) \cdot \cos(\varphi); \\
\rho(\varphi) &= \frac{a_f \cdot b_f}{\sqrt{b_f^2 \cdot \cos^2(\varphi) + \sqrt{+a_f^2 \cdot \sin^2(\varphi)}}}; \\
\psi(\varphi) &= \arctg \left( \frac{-b_f^2}{a_f^2 \cdot \tan(\varphi)} \right).
\end{align*}
\]

In the general case, the dependence \( R(h_b) \) cannot be obtained analytically, but the numerical solution of equations (18) – (21) is possible. As a result of the analysis of a mesh geometrical form and equations (18) – (21) it is possible to allocate two extreme FS steady positions 2 and 3 on fig. 4c, at which the minimum value of the radii of curvature FS is possible.

At position 2 (fig. 4c) FS is in the center of the hole and the radius of its curvature is identical to equation (12).

In position 3 (fig. 4c), the point of contact FS with the surface of the hole is in such position that the FS cross section is a semicircle. Then the radius of curvature FS is determined by the dependence:

\[
R = \frac{L_f}{2} - \frac{a_f \cdot b_f \cdot \cos(\varphi)}{\sqrt{b_f^2 \cdot \cos^2(\varphi) + \sqrt{+a_f^2 \cdot \sin^2(\varphi)}}}
\]

were \( \varphi = \arctg \left( \frac{-b_f^2}{a_f^2 \cdot \tan(\theta + \frac{\pi}{2})} \right) \).

In the industrial manufacture of wire metal mesh with square cells of plain and twill weaving [5], the mesh opening size is always larger than the wire diameter \( a_{ob} > d_w \). Then, taking \( a_c = a_{ob} \) and \( d_f = d_w \), the critical position can be considered such a position FS, which has the smallest value of the radius of curvature, which corresponds to the minimum value among equations (12) and (17) for a hole with round edges, and among equations (12) and (22) for the hole with elliptical edges.

Given the mathematical models and assumptions described above, we define mathematical models for calculating \( d_c \) of mesh cells.

In the case of the FS formation in the plain weave mesh (fig. 2a), the hole will have elliptical edges. Then the \( d_c \) of the mesh will be determined by dependence (10), where the principal radii of curvature are the minimum value obtained from formulas (12) and (22), substituting \( a_c = a_{ob} \sqrt{2}, a_f = d_w / \sqrt{2}, b_f = d_w \).

Wherein, if the contact angle \( \theta \) is equal to zero, then \( d_c \) for this scheme will be determined based on the dependence (4):

\[
d_c = a_{ob} \sqrt{2}.
\]

In the case of the formation of FS in the full-twill mesh (fig. 2b), a hole will have a more complex shape. The edges between the nearest wires will be round shape, and the edges between the far wires will be elliptical shape. Then the \( d_c \) of the mesh cell can be determined only by numerical calculation based on condition (11). The characteristic dimensions of such a cell are defined as \( a_1 \) (8) and \( a_2 \) (9). The function \( R_1(h_b) \) will be determined from (15) and (16), substituting \( a_c = a_{ob} \) and \( d_f = d_w \). The function \( R_2(h_b) \) will be determined from (18) – (21), substituting \( a_f = a_{dw}, b_f = b_{dw}, L_f = 2 \cdot a_1 + 2 \cdot d_w \), where:

\[
b_{dw} = \frac{d_w}{2 \cdot \cos(\beta_{dw})}.
\]

Wherein, if the contact angle is equal to zero, then \( d_c \) for this scheme also must determines by numerical simulation according to condition (11), where the main radii of curvature will have the form of functions (14):

\[
R_1(h_b) = \frac{h_b}{2} + \frac{a_{ob}}{8 \cdot h_b},
\]

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\[ R_2(h_b) = \frac{h_b}{2} + \frac{(2 \cdot a_{ob} + d_w)^2}{8 \cdot h_b}. \]  
(27)

To simplify the calculation for this scheme, it is possible to assume that the critical stable bubble height \( h_b = a_1/2 \). However, in this case we get slightly inflated values than in numerical simulations. Taking into account (14) the principal radii of FS curvature in equation (10) will have form:

\[ R_1 = \frac{a_1}{2}, \]  
(28)

\[ R_2 = \frac{h_b}{2} + \frac{a_2^2}{8 \cdot h_b} = \frac{1}{4} \left( a_1 + \frac{a_2^2}{a_1} \right). \]  
(29)

Taking into account (10), (28), (29) \( d_c \) holes of any shape will be determined by the equation:

\[ d_c = \frac{2}{a_1 + \frac{2}{a_1} + \frac{a_2^2}{a_1^2} \left( a_2 + \frac{a_2^2}{a_1^2} \right)}. \]  
(30)

Then, given (8), (9), (30), \( d_c \) of twill mesh cell may defines as:

\[ d_c = \frac{2}{a_{ob} + \frac{2}{a_{ob}} + \frac{(2 \cdot a_{ob} + d_w)^2}{a_{ob}}} \]  
(31)

**Scientific novelty**

The scientific novelty of the obtained results consists in the development of new mathematical models of FS formation in meshes with square cells of plain and twill weaving, characterized by taking into account the spatial structure of mesh weaving and curved wire shape, as well as FS formation on adjacent mesh cells with different contact angle. A simplified equation for determining the \( d_c \) of any shape cell for the case of complete wetting of the surface with liquid are obtained. As a result, the use of mathematical models developed in the current work allows increasing the accuracy of calculation of design characteristics of liquid acquisition devices of spacecraft.

**Conclusions**

Mathematical models for determining the bubble point pore diameter of meshes with square cells of micron sizes of plain and full-twill weave have been developed. The models takes into account the spatial structure of the real mesh weave and the factor of the contact angle influence between mesh surface with the working fluid. A mathematical model of an asymmetric mesh was developed, which is used to calculate the bubble point pore diameter of full-twill mesh.

Simplified mathematical models for the special case of complete wetting where the contact angle of liquid with mesh surface is zero have been developed.

The obtained mathematical models can be used in the design of capillary phase separators of liquid acquisition devices of spacecraft, as well as in the calculation of capillary devices of various technical systems, such as heat exchangers, gas-liquid mixers, chemical reactors, bubble filters and others.

Further research is possible in the areas of more detailed consideration of the spatial structure of mesh weaving, adaptation of mathematical models for the calculation of filter-type meshes, deformed meshes, and verification the calculation results using the obtained models by physical experiments.

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